Semester One

ATAR course examination, 2023

Question/Answer Booklet

MATHEMATICS
SPECIALIST - UNIT 1

Section Two:
Calculator-assumed

WA Student Number: In figures

In words   _____

Student Name   _____

Time allowed for this paper
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this paper
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	10	10	100	92	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2023: Part II Examinations.* Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator Assumed **(92 marks)**

This section has **10** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 **(9 marks)**

Given $x \in \mathbb{R}$, use the following statement to answer the questions below:

$$\text{If } x^2 + 5x < 0 \text{ then } x < 0$$

(a) (i) Write down the converse of this statement. (1 mark)

    (ii) Use a counter-example to prove the converse is false. (2 marks)

(b) (i) Write down the contrapositive of this statement. (1 mark)

    (ii) Prove the contrapositive is true. (2 marks)

(c) Hence, determine the truth of the original statement. Justify your answer. (2 marks)

(d) Explain why the statement $x^2 + 5x < 0 \Leftrightarrow x < 0$ is false. (1 mark)

Question 9   **(6 marks)**

The diagram below shows parallelogram $PQRS$, with $\overrightarrow{PQ} = \mathbf{q}$ and $\overrightarrow{PS} = \mathbf{p}$.
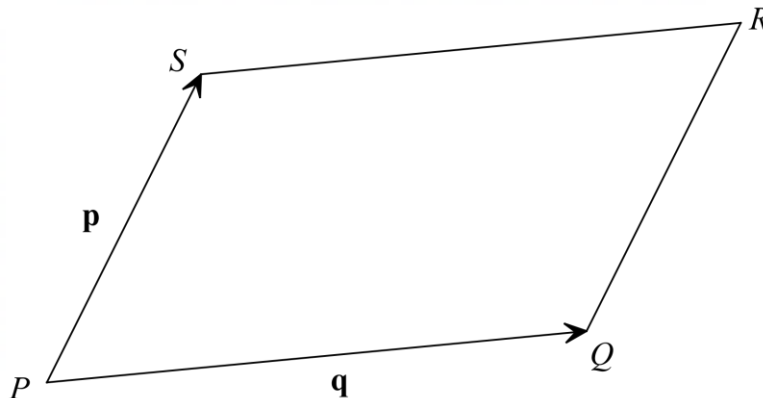
(a)   Write down a vector that represents $\overrightarrow{PR}$ into terms of $\mathbf{p}$ and $\mathbf{q}$.   (1 mark)

(b)   Write down a vector that represents $\overrightarrow{SQ}$ into terms of $\mathbf{p}$ and $\mathbf{q}$.   (1 mark)

(c)   Determine an expression for $\overrightarrow{PR} \bullet \overrightarrow{SQ}$.   (2 marks)

(d)   Describe geometrically shape $PQRS$, when $\overrightarrow{PR} \bullet \overrightarrow{SQ} = 0$, justifying your answer.   (2 marks)

Question 10   **(11 marks)**

(a) A bag contains six counters numbered 4, 5, 6, 7, 8 and 9. The counters are picked out, one at a time, and placed in a row from left to right.

    (i) How many six-digit numbers will have all the odd and all the even numbers together?   (2 marks)

    (ii) How many six-digit numbers will contain the digits 789 in that order?   (2 marks)

    (iii) How many six-digit numbers will **not** have the prime numbers adjacent?   (2 marks)

    (iv) If only three counters were picked out, one at a time, and placed in a row from left to right, how many of these numbers will have the digits in ascending order?   (1 mark)

(b) At a mathematics competition, there are 4 students from Alpha School, 7 from Beta College and 5 from Gamma School. Only 6 students can present their solutions, with each school picking 2 students.

How many different orders are possible,

    (i) if students are selected at random to present?   (2 marks)

    (ii) if students from the same school must present one after the other?   (2 marks)

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Question 11   **(13 marks)**

Vectors a and b are defined as $\mathbf{a} = -3\mathbf{i} + \mathbf{j}$ and $\mathbf{b} = 6\mathbf{i} - 6\mathbf{j}$.
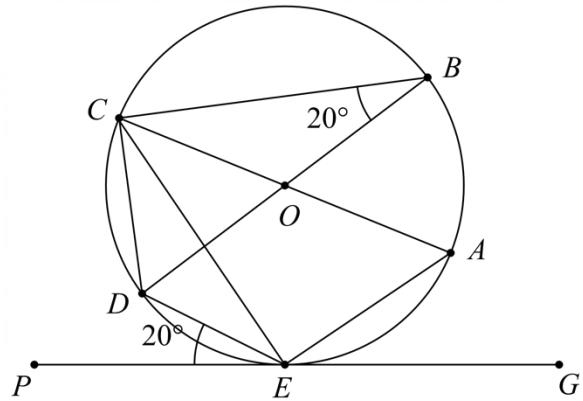
(a)   Determine $3\mathbf{a} - \mathbf{b}$.   (1 mark)

(b)   Determine the value of scalars x and y if $x\mathbf{a} + y\mathbf{b} = -36\mathbf{i} + 32\mathbf{j}$.   (2 marks)

(c)   Find a vector with the same length of $\mathbf{a}$ in the direction of $\mathbf{b}$.   (3 marks)

(d)   Determine the vector projection of $\mathbf{a}$ on $\mathbf{b}$.   (2 marks)

(e)   The vectors $\mathbf{b}$ and $\mathbf{c} = 3\mathbf{i} + x\mathbf{j}$ are of equal length. Determine x.   (2 marks)

(f)   Given $\mathbf{d} = 7\mathbf{i} + \mathbf{j}$, determine λ, such that $3\mathbf{a} + \lambda\mathbf{b}$ is perpendicular to $\mathbf{d}$.   (3 marks)

       See next page

Question 12 **(10 marks)**

In the diagram at right, A, B, C, D and E are five points on the circle with centre O.
AC and BD are diameters, and PEG is a tangent to the circle at E.

It is given that $\angle DEP = \angle DBC = 20°$.

(a) Determine, giving reasons, the size of the following angles:

(i) $\angle DEC$ (1 mark)

(ii) $\angle DCE$ (1 mark)

(b) Using part (a) and triangle CDE, explain why $\angle COD = \angle DOE$. (2 marks)

(c) Prove that $\angle EAC = 40°$, giving reasons. (3 marks)

(d) Is it possible to draw a circle through the points E, O, C and D. Justify your answer. (3 marks)

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Question 13   **(6 marks)**

Four dogs are arguing over a bone. The diagram shows the four dogs, the direction they are pulling in, and the force applied. The angle between King's force and Boston's force is 180°.
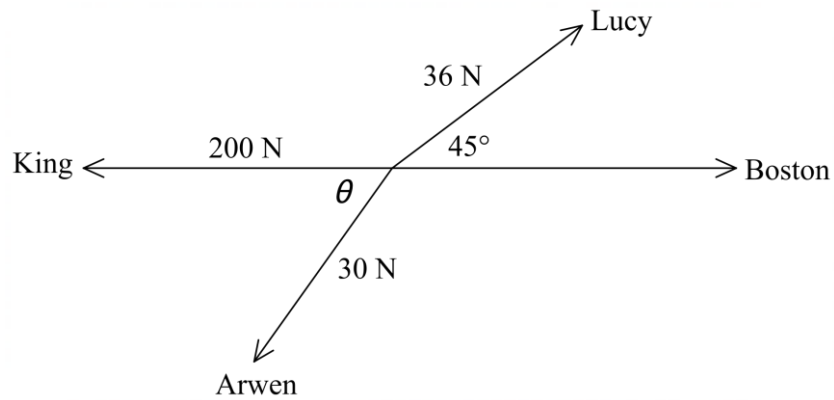
If the bone does not move, determine

(a)   the size of angle θ,   (3 marks)

(b)   the force that Boston pulls at.   (3 marks)

Question 14 **(11 marks)**

(a) Two students wrote in their notebooks an expression the teacher had written on the board.

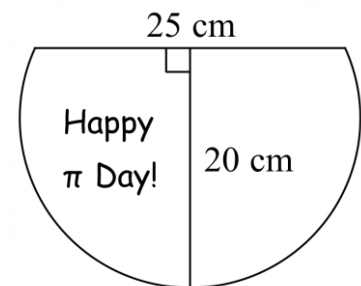Student 1 wrote: $\forall x \in \mathbb{R}(\exists y \in \mathbb{R}, x < y)$

Student 2 wrote: $\exists x \in \mathbb{R}(\forall y \in \mathbb{R}, x < y)$

One student copied down the expression incorrectly. If the teacher said the statement was true, who wrote it down incorrectly? Justify your answer. (3 marks)

(b) A teacher brought in cake for their class for Pi Day.

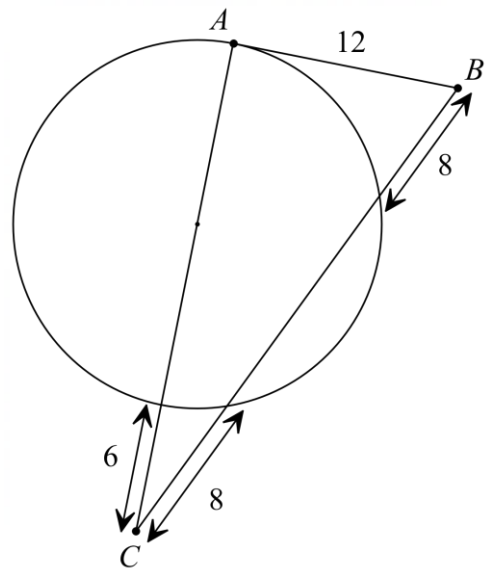Unfortunately, someone had sliced off a segment of cake and eaten it.

Use the given measurements to determine the diameter of the original cake, to one decimal place. (4 marks)

25 cm
Happy
π Day! 20 cm

(c) AB is a tangent to the circle given circle.

     Determine the area of ΔABC, correct to one decimal place.   (4 marks)

Question 15 **(11 marks)**

Two snails, Racer (R) and Speedy (S), are competing in a race.
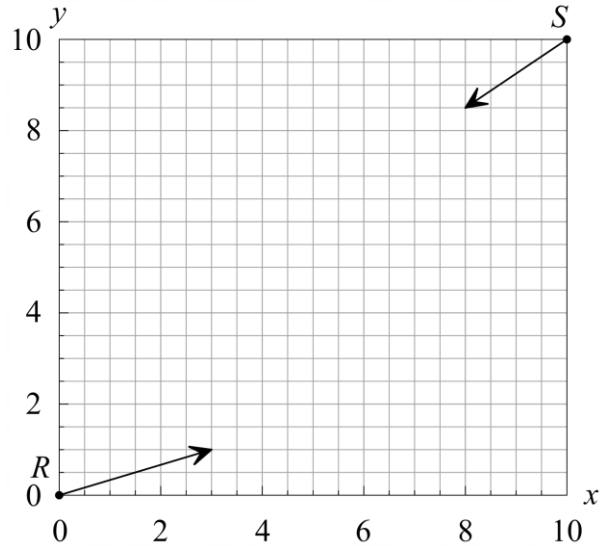
The snails start in opposite corners of a square of side length 10 cm, where $(0,0)$ represents the origin.

The initial position (in cm) and velocity vectors (in cms^{-1}) of the two snails are given by:

$$\mathbf{r}_R = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \mathbf{v}_R = \begin{pmatrix} 0.6 \\ 0.2 \end{pmatrix}$$
$$\mathbf{r}_S = \begin{pmatrix} 10 \\ 10 \end{pmatrix} \quad \mathbf{v}_S = \begin{pmatrix} -0.4 \\ -0.3 \end{pmatrix}$$

Let t represent the time in seconds since the snails left their respective corners.

The diagram above shows the starting positions, and the direction the snails are travelling in.

(a) Which snail is the fastest? Justify your answer using an appropriate calculation. (2 marks)

(b) Find the angle between the two snails' direction. (1 mark)

(c) Determine the position of each snail after t seconds. (2 marks)

The winner of the race is the first snail to reach the opposite corner.

Assuming the snails continue in the same direction until they hit the edge of the square, and then travel at the same speed towards the opposite corner,

(d)   determine which snail wins the race. (6 marks)

Question 16   **(4 marks)**

A bag consists of 10 cards, numbered 0 to 9 inclusive.

(a)   Show that if a student selects 7 of them, then there are two cards with a sum of 10.

(2 marks)

At least one card in the bag is now removed. The student does not know which card(s) have been removed.

(b)   The student now selects **6 cards**. Determine the **minimum** number of cards that can be removed, to guarantee at least two of the six chosen cards sum to 10. Justify your answer.
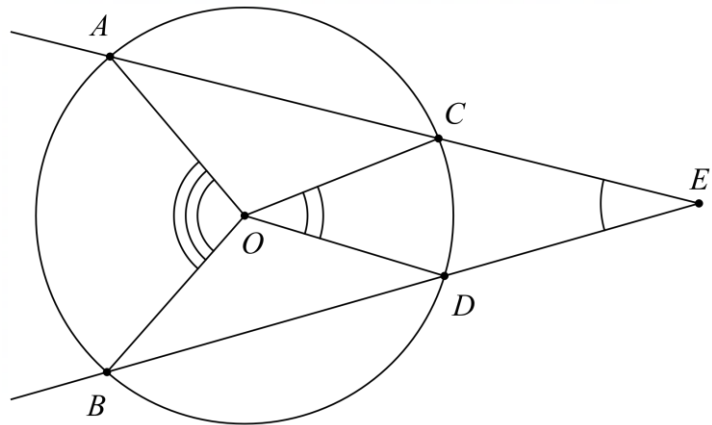
(2 marks)

Question 17 **(11 marks)**

The secant angle theorem states that the angle formed by two intersecting secants is half of the difference of the two interior angles.

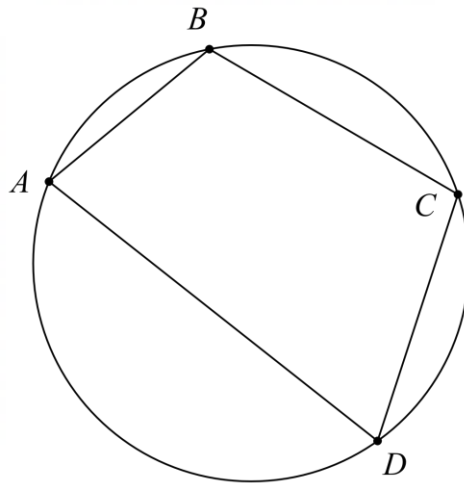This is

$$\angle CED = \frac{\angle AOB - \angle COD}{2}$$

(a) (i) Let $\angle AOB = 2\theta$, then state the size of angle $\angle ACB$. (1 mark)

(ii) Let $\angle COD = 2\phi$, then state the size of angle $\angle CBD$. (1 mark)

(iii) Using ΔECB and parts (i) and (ii), prove the secant angle theorem. (3 marks)

See next page

In the diagram below $ABCD$ is a quadrilateral with $\angle ABC + \angle CDA = 180°$.

(b)   Use the secant angle theorem from part (a), and proof by contradiction to prove that $ABCD$ is cyclic.

     Hint: Prove that $\angle BCD$ must lie on the circumference.   (6 marks)

End of questions